#13: Triangle Laws

December 6, 2008

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the non-right triangles were getting sad

The final installment of 2008 is also the final installment in our study of trigonometry (but never fear, there will still be plenty of triginometry involved in future assignments)! You already know how trig functions relate to *right* triangles; this week, we will study two fundamental *triangle laws* that relate sine and cosine to *all* triangles. After all, this is *trigonometry*, ("triangle measurement"), not *orthotrigonometry* ("right triangle measurement")!

1 The Law of Sines

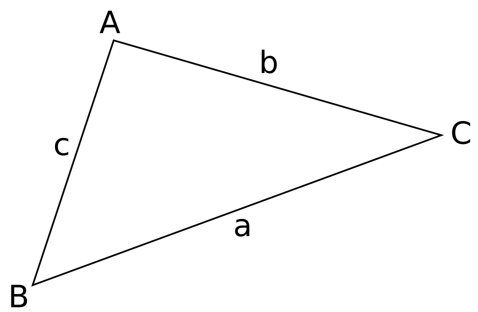

Figure 1: A triangle.

Figure 1 shows a generic triangle ABC. Notice how it is labeled: the side opposite angle A is labeled a; the side opposite angle B is labeled b, and so on. We will use the capital letters to refer both to the vertices of the triangle and to the angles at those vertices.

the Law of Sines

The law of sines says this:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \tag{1}$$

In words, for any triangle ABC, the ratios between the sines of the angles and the corresponding opposite side lengths are all the same. Note that this ratio may be different in different triangles. The Law of Sines just says that in any particular triangle, all three ratios will be the same.[1]

Problem 1. Check that the Law of Sines holds for the right triangle shown in Figure 2. Show your work.

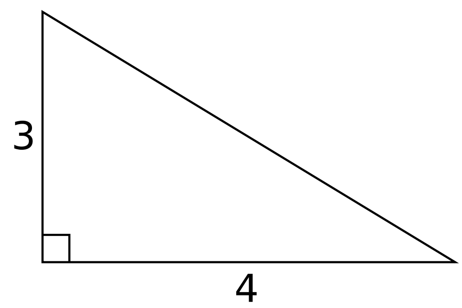

Figure 2: A 3-4-5 right triangle.

Problem 2. Consider the triangle shown in Figure 3.

(a) What is θ?

(b) What is the length of side q? Show your work.

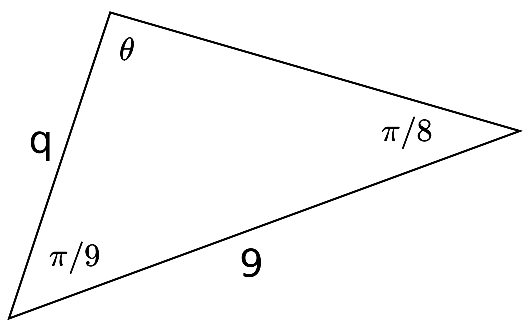

Figure 3: A triangle with given angle, side, angle (ASA)

You can also read about something interesting I discovered about the Law of Sines while writing this assignment here: `http://www.mathlesstraveled.com/?p=194`.

[1] In fact, the ratios are equal to the diameter of the triangle's circumscribed circle! You probably don't need to know that, but it is nifty.

2 The Law of Cosines

the Law of Cosines

Again referring to Figure 1, the Law of Cosines says that

$$c^2 = a^2 + b^2 - 2ab\cos C. \tag{2}$$

Problem 3. Explain the Law of Cosines in words.

Problem 4. What happens to the Law of Cosines when C is a right angle?

Problem 5. Find the length of side h of the triangle shown in Figure 4. Show your work. Round your answer to three decimal places.

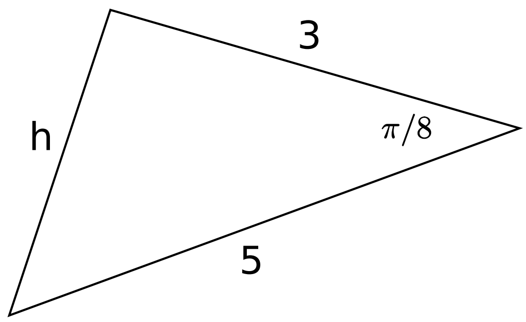

Figure 4: A triangle with given side, angle, side (SAS)

Problem 6. In the triangle shown in Figure 5, find angle θ. Show your work. Round your answer to three decimal places.

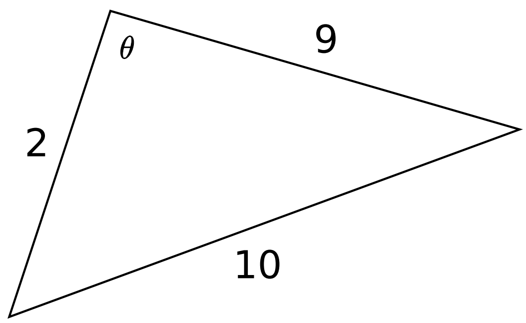

Figure 5: A triangle with three given sides (SSS)

Problem 7. In the triangle shown in Figure 6, find angle θ. Show your work. What does it mean that you get two answers? Is this reasonable?

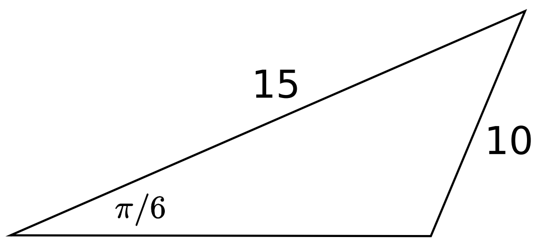

Figure 6: A triangle with given angle, side, side (. . .)

