# #29: Logarithm review

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*review?*

This week we're going to spend some time reviewing logarithms. I say "review" since you've probably seen them before in theory, but if my experience is any guide, it's quite likely that you've forgotten most of what you used to know about them!

*why students dislike logarithms*

In my experience, most students seem to really despise logarithms. I'm not entirely sure why, but I have a few guesses:

1. Learning about logarithms often seems to consist of just learning a bunch of (seemingly) arbitrary rules, and using them to solve (tedious, uninteresting) problems.

2. They never seem to come up in any other topic—they're just like some isolated topic that you learn for no good reason, and never hear about again!

3. The arbitrary rules alluded to in item (1) are confusing and difficult to remember.

**Problem 1.** Sound familiar? With which of these items do you agree? Are there any other reasons? (If you think logarithms are happy fun times, it's OK to say that too.)

*the real scoop*

I also have some responses to these assertions:

1. There's actually only *one* rule that you need to know—all the other rules follow from it, if you understand that one rule really well. I can't really argue about the tediousness of the sorts of problems that use logarithms, unfortunately—but see the next response.

2. It turns out that logarithms come up *all the time*, in some very fundamental ways, in the study of calculus. Now, there *used* to be a very good reason to learn about logarithms long before you got to calculus—which I will explain later. The problem is that *this reason no longer exists*, so no wonder students feel like they are an isolated topic with no relation to anything else—because until you get to calculus, it's true! So I hope you can trust me when I say: understanding

 

logarithms *will* be useful, eventually, and if you feel like they seem kind of pointless now, you are not wrong.

Logarithms (specifically, $\log_2$) also come up a lot in the branch of computer science that studies algorithmic complexity.

3. Arbitrary rules are only difficult to remember if you don't use them a lot. See point (2).

With that out of the way—onwards!

## 1 Logarithms and exponents

*the only thing you need to know*

Here is the most important—in some sense, the *only*—thing you need to know: logarithms are the opposite of exponents! More specifically:

***logarithms***

If

$$b^e = a$$

then

$$\log_b a = e.$$

(Note: to typeset logarithms in LaTeX, use `\log`: for example, `\log_6 (q+1)` renders as $\log_6(q+1)$.

Put another way:

$$b^e = ?$$

asks, "if you multiply $b$ by itself $e$ times, what do you get?" and

$$\log_b a = ?$$

*translation exercise*

asks, "how many times do you have to multiply $b$ by itself to get $a$?"

**Problem 2.** Translate each exponential equation to an equivalent one using logarithms, and vice versa.

(a) $2^8 = 256$

(b) $3 = \log_b 125$

 

(c) $z = x^{1024}$

(d) $\log_8 q = f$

**Problem 3.** Evaluate:

(a) $\log_2 16$

(b) $\log_5 5$

(c) $\log_4 64$

(d) $\log_9 3$

(e) $\log_7 1$

*some special logarithms*

**Problem 4.** Suppose $b > 1$.

(a) What is $\log_b b$?

(b) What is $\log_b 1$?

(c) What is $\log_b b^e$?

(d) What is $b^{\log_b e}$?

## 2 Logarithm rules

*logarithm rules*

There are three main rules specifying how logarithms can be manipulated. However, each of them is a direct consequence of the definition of logarithms from the previous section (as the inverse of exponentiation). Let's see if you can figure them out.

*the multiplication/addition rule*

**Problem 5.** Consider $\log_b(xy)$.

(a) I claim that $x = b^{\log_b x}$. Why is this?

(b) Of course, $y = b^{\log_b y}$ as well. Substitute these two expressions for $x$ and $y$ in the expression $\log_b(xy)$. What do you get?

(c) Can you simplify the resulting expression, using the laws of exponents?

 

(d) Can you simplify the result again, using what you know about logarithms? (Hint: see Problem 4. . . )

(e) What logarithm law have you discovered?

In English, this law says that *the logarithm of a product is the sum of the logarithms*. In other words, logarithms turn multiplication into addition!

*the division/subtraction rule*

It is likewise true (although I won't make you show this one; it is quite similar to Problem 5) that logarithms turn division into subtraction:

$$\log_b(x/y) = \log_b x - \log_b y.$$

*This* is why logarithms were once useful outside of calculus: *adding* is a lot easier than *multiplying*, so logarithms could be used to help perform multiplication much more quickly. Here's how it worked: say you wanted to multiply $x$ and $y$, which are too big to easily multiply by hand.[1] So you get out your handy Table O' Logarithms[2] and look up the logarithms of $x$ and $y$. Then you add those (which is pretty easy) and get $\log_e x + \log_e y$ (note that most Tables O' Logarithms were to the base $e$). But $\log_e x + \log_e y = \log_e(xy)$, so now you take this number and do a reverse lookup (in the second half of your Enormous Book—kind of like a bilingual dictionary) to see what it is the logarithm *of*, and of course you get $xy$.

**Problem 6.** Why doesn't anyone have an Enormous Book O' Logarithms anymore?

**Problem 7.** This same idea was the basis for *slide rules*. Look up slide rules on the Internet (Wikipedia is a good starting place, but also try following some of the "related links" at the bottom of the page) and explain what they were, how you used them, and why no one really uses them anymore.

**Problem 8.** If logarithms turn multiplication into addition, then they turn exponentiation into. . . what? (*Hint:* think about $\log_b(x^a)$. What does $x^a$ mean? Can you apply the multiplication-to-addition rule?)

**Problem 9.** Simplify.

(a) $\log_2(4^7)$

[1] Note, when I say *big*, I really just mean *having a lot of decimal places*: it is just as tedious to multiply 1.23456789 by 89.362349763 as it is to multiply 123456789 by 89362349763.

[2] By which I mean Enormous Book O' Logarithms.

 

(b) $\log_3(x^8 9^2)$

(c) If $\log_b 3 = 1.4$, what is $\log_b 27$?

(d) If $\log_b 5 = x$ and $\log_b 3 = y$, what is $\log_b 225$?

*change-of-base formula*

And now for the final rule: the *change-of-base* formula.

$$\log_a x = \frac{\log_b x}{\log_b a}.$$

This says that the logarithm to base $a$ of $x$ is the same as the logarithm to base $b$ of $x$, divided by the logarithm to base $b$ of $a$. This is a very useful formula to know for evaluating logarithms on your graphing calculator, since it can only do logarithms base 10 and base $e$; you can use the change-of-base formula to evaluate a logarithm to any base $a$ as long as you can evaluate logarithms to some particular base $b$ (with your calculator, $b = 10$ or $e$).

**Problem 10.** You graphing calculator has two buttons for performing logarithms. The button labelled "log" does $\log_{10}$. The button labelled "ln" (which is an abbreviation for "natural logarithm" (probably in French or something)) does $\log_e$. What is $e$? Well, it's approximately $2.71828\ldots$ but you'll have to wait until calculus to find out why it's so special!

Use your graphing calculator to evaluate each of the following. Round your answers to three decimal places.[3]

(a) $\log_2 50$

(b) $\log_{10} 200$

(c) $\log_9 27$

**Problem 11.** Use your graphing calculator to make a graph of $y = \ln x$. Describe the graph. Give as much detail as possible.

**Problem 12.** Solve for $x$.

(a) $2^{x+5} = 4^x$

(b) $5^{x-3} = 17$

(c) $\log_7(3x) = 5$

(d) $5^x = 3^{2x+1}$

[3] Or whatever.

 