Precalculus Syllabus

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2008–2009
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1 Schedule of topics

Note: this schedule can and probably will change slightly!

The year will be divided into three main units. The first unit will cover some of the foundations of modern mathematics in depth, and allow for some review of topics from previous years. The remaining two units will correspond to the two major threads of mathematics over the last several hundred years—the continuous and the discrete. Continuous mathematics concerns itself with measurement, motion, and change; discrete, with counting, pattern, and structure.

Week	Date	Topic
		Foundations
1	9/2–5	LaTeX introduction, set theory I
2	9/8–12	Functions
3	9/15–19	Numbers
4	9/22–26	Set theory II
5	9/29–10/3	Logic, proof, and problem-solving I
6	10/6–10	Logic, proof, and problem-solving II
7	10/13–17	Interlude: Logic circuits
8	10/20–24	Algebra/geometry review
		Continuous
9	10/27–31	Foundations of trigonometry I

Week	Date	Topic
10	11/3–7	Foundations of trigonometry II
11	11/10–14	Inverse trigonometric functions
12	11/17–21	Trigonometric identities
	11/24–28	*Thanksgiving*
13	12/1–5	Triangle laws
14	12/8–12	Interlude: Acoustics
	12/15–1/4	*Christmas/New Year*
15	1/5–9	Matrices
16	1/12–16	Rectangular and polar coordinates
17	1/19–23	Vectors and vector functions
18	1/26–30	Complex numbers
19	2/2–6	Interlude: Fractals and computer graphics
		Discrete
20	2/9–13	Combinatorics and probability I
21	2/16–20	Combinatorics and probability II
22	2/23–27	Sequences
23	3/2–6	Series I
24	3/9–13	Series II
25	3/16–20	Induction and recursion
26	3/23–27	Number Theory I
27	3/30–4/3	Number Theory II
28	4/6–10	Interlude: Cryptography
	4/13–17	*Easter*
29	4/20–24	Group Theory I
30	4/27–5/1	Group Theory II
31	5/4–8	TBD

Week	Date	Topic
32	5/11–15	TBD
33	5/18–22	Overflow
34	5/25–29	Overflow
35	6/1–5	Revise final assignment and wrap-up

2 Format

You will receive weekly assignments in the form of PDF documents containing instruction mixed with problems for you to solve. You will receive each new assignment at the beginning of the week (exact day and time to be determined), and your solutions to the problems will be due exactly one week after receiving the assignment.

I will provide feedback on each submitted solution set no more than two days after it is turned in. You will then have the remainder of the week to revise the solution set, which will be due at the same time as your solutions to the next assignment. In all, this means that each Monday (or whatever day we choose), you will:

- turn in a revised version of assignment $n - 1$
- turn in assignment n
- receive assignment $n + 1$

So, the first few weeks might look something like this, assuming that Monday is the day we choose as the day to receive new assignments:

Tuesday, 9/2	Receive assignment 1 (on Tuesday instead of Monday due to Labor Day)
Monday, 9/8	Receive assignment 2
Tuesday, 9/9	Assignment 1 due (1 week after assigned)
by Thursday, 9/11	Receive feedback on assignment 1
Monday, 9/15	Assignment 2 and revised assignment 1 due; receive assignment 3
by Wednesday, 9/17	Receive feedback on assignment 2
Monday, 9/22	Assignment 3 and revised assignment 2 due; receive assignment 4
etc.	

Our primary method of communication will be by emailing PDF documents prepared using the free LaTeX typesetting system (which you will learn about in the first week's assignment). There are also several other free tools we can use to communicate. A simple tool for quick questions and discussions is Gmail Chat. For discussions that require writing mathematical formulas, we can use MathIM (`http://www.mathim.com/`) which allows LaTeX formulas to be embedded in chat. Finally, for discussions which require drawing pictures, we can use ScribLink (`http://www.scriblink.com/`).

3 Expectations

Submitted solutions should be:

- **complete**: if you are stuck on some of the problems, you are expected to ask me for help or hints during the week. It is not acceptable to turn in an assignment and write "we didn't know how to solve this problem" if you have not asked for help. (It is, however, perfectly fine if you have asked for help and just still don't get it; the important point is to ask for help.) Of course, a corollary is that you should *not* wait until just before an assignment is due to work on it! I suggest taking the beginning of the week to solve all the problems on paper, and the last few days to type up your solutions and work out any last-minute details.

- **well-written**: check your grammar and spelling, and make sure your solutions are clear. Math that exists only in someone's head is like a tree falling in a forest with no one around to hear it—communicating well is a fundamental mathematical skill, not an afterthought.

- **concise**: your solutions should be as simple and elegant as possible. I *don't* want to see every single step! If a step or a detail is obvious to you, omit it. For example, it would be perfectly acceptable to write, "this results in the equation $2x^2 - 6 = 2x + x^2$, which has solutions $x = 1 \pm \sqrt{7}$"; you wouldn't need to show the process of actually solving the equation, since it is obvious to anyone who knows some algebra.

- **on time**: solution sets should be turned in on time. If there is a good reason you would like some extra time on an assignment (a trip, some sort of emergency, an exceptionally large workload in other classes), just ask. However, you must ask at least *24 hours in advance*—I will not grant retroactive extensions, except in the case of emergencies. See below for details on the grading of late assignments.

- In addition, at the end of each submitted solution set, you should include a section with comments on the problem set. What was particularly interesting or uninteresting? What was too easy, too hard, or just right? Was it too long? Too short? Were any parts confusing? How long did you spend on it? What would you like to learn more about? These particular questions are suggestions, not a checklist of questions to answer; feel free to write down any comments or questions you might have. The important point is to provide me with some "meta-feedback" about the assignments so I have an idea of what direction to take future assignments.

- Finally, note that when submitting your solutions to an assignment, you should submit *both* the LaTeX source (`.tex`) file *and* the generated PDF.

4 Grading

Each submitted assignment will be graded on a scale of five points. The scale runs approximately as follows:

- **5** — the solutions are complete, correct, clear and concise. There may be a few minor grammatical or spelling errors, but no mathematical errors.
- **4** — the solutions are generally correct and clear, but may have some minor errors and/or unclear portions. Some sections may need to be rewritten to be made clearer or simpler.
- **3** — most solutions are generally correct, but there may be significant portions which are incorrect, unclear, or overly complex. One or more solutions may have major gaps in logic or use an incorrect approach. No more than two solutions are missing entirely.
- **2** — many solutions are incorrect, incomplete, or missing. Solutions are unclear and need substantial rewriting.
- **1** — the assignment was attempted, but is largely incorrect or incomplete, or it is so poorly written and confusing that no one can tell if it is correct or not.
- **0** — the assignment was not turned in at all, had no real content, or was turned in more than 48 hours late.

Fractional scores may be awarded. Of course, this scale is quite subjective, but I will try to be as consistent and fair as possible. If you ever have questions about scoring you are of course free to ask.

Keep in mind that I will provide feedback on the first version of an assignment, with the expectation that you should be able to incorporate the feedback in order to receive a 5 (or very close to it) on the revised version of the assignment. Your final score for a given assignment will be a weighted average of your scores on the first draft and the revised version, with the revised version counting triple. For example, if you received a 3 on a first draft, and revised it to receive a 5, your final score would be $(3 + 3 * 5)/4 = 4.5$.

Late assignments will be penalized by a number of points $p(t)$, where t is the number of hours by which the assignment is late, and

$$p(t) = \frac{1}{3}(2^{t/12} - 1).$$

Thus, an assignment turned in 24 hours late will be penalized by $p(24) = 1$ point; an assignment turned in 48 hours late will be penalized by $p(48) = 5$ points (thus receiving a score of zero no matter what score it would have received otherwise; assignments cannot receive negative scores). The purpose of this system is to allow for reasonable flexibility while still providing firm consequences for lateness. For example, it is well worth it to turn in an assignment one hour late (incurring only a $p(1) \approx 0.02$ point penalty, hardly noticeable) if it gives you time to improve it a bit. It is probably worth it to turn in an assignment twelve hours late (incurring a $p(12) = 1/3$ point penalty) if it gives you time to improve it substantially or finish the last few problems. It probably isn't worth it to turn in an assignment 24 hours late (1 point penalty), or 30 hours late (1.552 points), or 36 hours late (2.333 points)... as you can see, the exponential scale leads to increasingly dire consequences.

5 Resources

Here are a few resources which you may find helpful throughout the year:

- Wolfram MathWorld (`http://mathworld.wolfram.com`) is a fantastic site with tons of reference material; essentially a comprehensive, free mathematics encyclopedia. Because of its aim of comprehensiveness and its encyclopedia-like nature, however, the articles can sometimes be quite terse and hard to follow.
- Wikipedia (`http://en.wikipedia.org`) actually has an excellent, and growing, set of mathematics articles. They are often a bit more descriptive and easier to follow than the articles on MathWorld.
- The Online Encyclopedia of Integer Sequences (`http://www.research.att.com/~njas/sequences/`) is exactly what its name says—an encyclopedia containing almost 150,000 integer sequences, with data, descriptions, references, and relationships for each! It might sound silly, but it's actually pretty cool.
- The Math Less Traveled (`http://www.mathlesstraveled.com/`) is my mathematics blog. I'd encourage you to follow it during the year—it's quite likely I'll be writing about some topics inspired by our course.

- Ask Dr. Math (`http://mathforum.org/dr.math/`) has a large collection of math questions and answers.
- Check the website for more resources!