

CSci 360, Fall 2004, Assignment 7

This assignment, worth 40 points, is due Friday, October 29, at the beginning of class. You should submit a paper copy of your lambda expressions to my office or at the beginning of class.

You may find *Lambda Calculator*, already installed on the Linux computers, useful in performing the following. (*Lambda Calculator* is also available for download from www.cburch.com/proj/lambda/.) To use it, first retrieve a file called “math.lmb” from the class Web site containing the definitions described on this handout. Then start *Lambda Calculator* by typing the following at the command line.

```
unix% lambda-calc math.lmb &
```

Important: You may *not* use the Y combinator defined in class (which we used in defining the factorial function).

1. Recall our lambda expressions for representing Boolean values.

$$\begin{aligned}\text{true} &= \lambda x.\lambda y.x \\ \text{false} &= \lambda x.\lambda y.y\end{aligned}$$

Write a lambda expression for computing the AND of two Boolean values. That is, with your definition in hand, I should be able to test it with the following.

expression	result
and false false	false
and false true	false
and true false	false
and true true	true

2. Recall our lambda expressions for Church numerals

$$\begin{aligned}0 &= \lambda s.\lambda z.z \\ 1 &= \lambda s.\lambda z.s z \\ 2 &= \lambda s.\lambda z.s (s z) \\ 3 &= \lambda s.\lambda z.s (s (s z)) \\ \text{inc} &= \lambda a.\lambda s.\lambda z.s (a s z) \\ \text{dec} &= \lambda a.\lambda s.\lambda z.a (\lambda f.\lambda g.g (f s)) (\lambda x.z) (\lambda x.x) \\ + &= \lambda a.\lambda b.\lambda s.\lambda z.a s (b s z) \\ \text{is0} &= \lambda a.a (\lambda x.\text{false}) \text{true}\end{aligned}$$

Based on these, write a lambda expression for computing the difference of two numbers. For example, if $-$ represents your expression, $- 5 2$ should return 3. Do not worry about what happens when the result should be negative.

3. Write a lambda expression for determining whether one number is less than or equal to another; For example, if \leq represents your expression, $\leq 5 2$ should return `false`, while $\leq 2 5$ should return `true`. (**Hint:** See what happens when you subtract one number from a smaller number (when the result ought to be negative).)
4. Write a lambda expression for determining whether one number is equal to another.

When you write your lambda expressions on what you submit, please substitute symbols for subexpressions when it makes the expression's meaning easier to understand. For example, write 2 rather than $\lambda s.\lambda z.s (s z)$ when appropriate.