Question 7.3–1: (Solution, p 3) Suppose we have a perceptron with three inputs, and the perceptron's current weights are $\langle 0.5, 0.2, -0.5 \rangle$.

a. What would the perceptron predict given the input $\langle 1, -1, -1 \rangle$?

b. Suppose this prediction were wrong. How would the perceptron update its weights if the learning rate were $r = 0.1$?

Question 7.3–2: (Solution, p 3) Suppose we have a three-input perceptron, where input x_1 is permanently wired to a 1 input.

a. Suppose the weights of the perceptron were $\langle 0.1, 0.5, -0.4 \rangle$, and we were to give it the inputs $\langle 1, 1, 1 \rangle$. What would the perceptron output?

b. Suppose we wanted to select weights so that our perceptron would behave like the AND of the inputs x_2 and x_3.

x_1	x_2	x_3	o
1	−1	−1	−1
1	−1	1	−1
1	1	−1	−1
1	1	1	1

Give a combination of perceptron weights w_1, w_2, and w_3 that would result in the desired behavior.

Question 8.1–1: (Solution, p 3) Consider the following context-free grammar.

$$\mathbf{S} \rightarrow \varepsilon \mid a\,\mathbf{S}\,b\,\mathbf{S} \mid b\,\mathbf{S}\,a\,\mathbf{S}$$

Give a derivation of the string *aabbab* using this grammar.

Question 8.1–2: (Solution, p 3) Consider the following context-free grammar.

$$\begin{aligned} \mathbf{S} &\rightarrow \mathbf{V}\,\mathbf{V} \mid \mathbf{V} \\ \mathbf{V} &\rightarrow (\,\mathbf{S}\,) \mid x \mid y \mid z \end{aligned}$$

Each of the following sentences is either described by this grammar or not. If it is, give either a parse tree with **S** at its root or a derivation from **S**. If it is not, simply say so.

a. *x (y)*

b. *x y z*

c. *x (y z)*

Question 8.1–3: (Solution, p 3) Write a context-free grammar describing each of the following languages.

a. the set of strings of *a*'s and *b*'s beginning and ending with the letter *a*

b. the set of strings containing either only *a*'s or only *b*'s

c. the set of strings of left-brackets and right-brackets where the brackets match (you can pair all brackets so that each pair includes a left bracket and a right bracket occurring after it in the string). Examples include *[[]][]* and *[[[]]]*.

Question 8.2–1: (Solution, p 4) Write a regular expression describing each of the following languages.

a. strings containing only *a*'s and *b*'s where all *a*'s come before all *b*'s.

b. strings containing only *a*'s and *b*'s in which "*aab*" somewhere occurs as an adjacent sequence (like *aabaa* or *baaba* but not *abbab*).

c. binary representations of positive even numbers.

Question 8.2–2: (Solution, p 4) Give a regular expression for each of the following languages.

a. all strings containing an even number of *a*'s (and no other letters).

b. all strings that contain either only *a*'s or only *b*'s.

c. all representations of binary numbers that are at least 4.

Question 8.2–3: (Solution, p 4) Give an English description of a language that can be described by a context-free grammar but not by a regular expression.

Solution 7.3–1: (Question, p 1)

a. It would predict 1.

b. $\langle 0.4, 0.3, -0.4 \rangle$

Solution 7.3–2: (Question, p 1)

a. 1 (The weighted sum is 0.2, which exceeds 0.)

b. $\langle -0.5, 1, 1 \rangle$ (There are many other correct answers.)

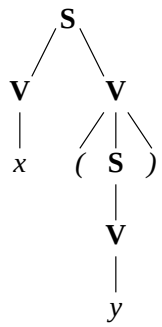
Solution 8.1–1: (Question, p 1)

$$\begin{aligned}
\mathbf{S} &\Rightarrow a\mathbf{S}b\mathbf{S} \\
&\Rightarrow aa\mathbf{S}b\mathbf{S}b\mathbf{S} \\
&\Rightarrow aab\mathbf{S}b\mathbf{S} \\
&\Rightarrow aabb\mathbf{S}a\mathbf{S}b\mathbf{S} \\
&\Rightarrow aabba\mathbf{S}b\mathbf{S} \\
&\Rightarrow aabbab\mathbf{S} \\
&\Rightarrow aabbab
\end{aligned}$$

[Note that it is important in a derivation to replace exactly one symbol in each step. Do not combine steps.]

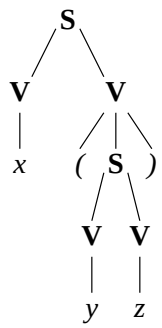
Solution 8.1–2: (Question, p 1)

a.



b. The sequence is not described by the grammar

c.



Solution 8.1–3: (Question, p 1)

a.

$$\begin{aligned} \mathbf{S} &\rightarrow a\,\mathbf{T}\,a \\ \mathbf{T} &\rightarrow a\,\mathbf{T} \mid b\,\mathbf{T} \mid \varepsilon \end{aligned}$$

b.

$$\begin{aligned} \mathbf{S} &\rightarrow \mathbf{A} \mid \mathbf{B} \\ \mathbf{A} &\rightarrow a\,\mathbf{A} \mid \varepsilon \\ \mathbf{B} &\rightarrow b\,\mathbf{B} \mid \varepsilon \end{aligned}$$

c.

$$\mathbf{S} \rightarrow \mathbf{S}\,\mathbf{S} \mid [\,\mathbf{S}\,] \mid \varepsilon$$

Solution 8.2–1: (Question, p 2)

a. `a*b*`

b. `(a|b)*aab(a|b)*`

c. `1(0|1)*0`

Solution 8.2–2: (Question, p 2)

a. `(aa)*`

b. `a*|b*`

c. `1(0|1)(0|1)(0|1)*` [Binary numbers that are at least 4 must have at least three bits. This regular expression describes all combinations of 0's and 1's that begin with a 1, have two more bits, and can have any number of bits thereafter.]

Solution 8.2–3: (Question, p 2) The language of palindromes containing *a*'s and *b*'s is a context-free language, but it is not a regular language.