Name: __________________________

1. [10 pts] Design a context-free grammar for the language of all strings containing only $a$’s and $b$’s with at least one $a$ and at least one $b$.

2. [10 pts] Design a finite automaton that accepts the language of all strings that contain $abaa$ (such as $abababa$ or $aabhaaa$). You need not worry about strings containing letters other than $a$’s and $b$’s.

3. [10 pts] Consider the following Turing machine. (Note that the underscore represents a blank on the tape.)

```
\[ \begin{array}{c|c}
0 & a, b \\
1 & b, a < \\
2 & a, b > \\
\end{array} \]
```

At right, diagram this Turing machine’s computation as it goes through the string $ababb$. If you run out of blanks in the table, stop.

To represent the machine’s initial position in the table at right, we write “$0 \ ababb$”. This represents a tape containing “$ababb$” (with blanks extending infinitely both ways), where the Turing machine is currently in state 0 of its finite automaton, and its head is pointing to the initial $a$. 

```
Solutions, Quiz 7, CSCI 150, Fall 2003

Statistics

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>23.773 (523.000/22)</td>
</tr>
<tr>
<td>stddev</td>
<td>5.107</td>
</tr>
<tr>
<td>median</td>
<td>23.500</td>
</tr>
<tr>
<td>midrange</td>
<td>19.000-28.000</td>
</tr>
</tbody>
</table>

#1. 4.50 / 10

#2. 7.14 / 10

#3. 8.14 / 10

+ 4-point bonus

1.

\[ S \to Ta \ T b T | T b T a T \]

\[ T \to a T | b T | \varepsilon \]

2.

3.

(The at this point, the machine has nowhere to go, and so it stops.)