Based on August 30 material

- §1.5 (p 73): 4
- If \( x^2 \) is irrational, then \( x \) is irrational.
  a. Prove it.
  b. What kind of proof did you use for the previous problem (direct, indirect, vacuous, trivial, contradiction)?
- Is the product of two irrational numbers always rational, always irrational, or sometimes rational and sometimes not? Prove your answer.
- Suppose \( n \) is a three-digit number — i.e., \( n = 100a + 10b + c \) for three integers \( a, b, \) and \( c \) between 0 and 9. Prove that \( n \) is a multiple of 3 if and only if the sum of \( n \)'s digits (i.e., \( a + b + c \)) is a multiple of 3.

Based on September 1 material

- §2.4 (p 167): 14. The answer is 24, but argue that this is the answer without resorting to a calculator or computer.
- §2.4 (p 167): 26 (\( \phi \) is defined in 25)
- §2.4 (p 168): 46. This is implied if you can show that any divisor of both \( a \) and \( m \) also is also a divisor of both \( b \) and \( m \). (You don’t need to mention this, but the symmetric argument will also show that any divisor of \( b \) and \( m \) is also a divisor of \( a \) and \( m \). As a result, the two sets of divisors are the same, and so the greatest in each set is the same.)
- The previous problem leads to the Euclidean algorithm for computing greatest common divisors: The GCD of \( a \) and 0 is \( a \), while the GCD for \( a \) and a non-zero \( b \) can be computed by computing the GCD of \( a \mod b \) and \( b \).
  a. Use this algorithm to find the following GCDs, showing your intermediate steps:
     i. \( \gcd(40, 30) \)
     ii. \( \gcd(84, 60) \)
  b. Explain why the Euclidean algorithm is related to the previous problem.