**Question 1** (K&T 8.4)**.** Suppose you're consulting for a group that manages a high-performance real-time system in which asynchronous processes make use of shared resources. Thus the system has a set of $n$ *processes* and a set of $m$ *resources*. At any given point in time, each process specifies a set of resources that it requests to use. Each resource might be requested by many processes at once; but it can only be used by a single process at a time. Your job is to allocate resources to processes that request them. If a process is allocated all the resources it requests, then it is *active*; otherwise it is *blocked*. You want to perform the allocation so that as many processes as possible are active. Thus we phrase the RESOURCE RESERVATION problem as follows: Given a set of processes and resources, the set of requested resources for each process, and a number $k$, is it possible to allocate resources to processes so that at least $k$ processes will be active?

Consider the following list of problems. For each problem either give a polynomial-time algorithm or prove that the problem is NP-complete.

- The general RESOURCE RESERVATION problem defined above.
- The special case of the problem when $k = 2$.
- The special case of the problem when there are two types of resources—say, people and equipment—and each process requires at most one resource of each type. (In other words, each process requires at most one specific person and one specific piece of equipment.)
- The special case of the problem when each resource is requested by at most two processes.

**Question 2** (K&T 8.5)**.** Consider a set $A = \{a_1, \dots, a_n\}$ and a collection $B_1, B_2, \dots, B_m$ of subsets of $A$ (*i.e.*, $B_i \subseteq A$ for each $i$).

We say that a set $H \subseteq A$ is a *hitting set* for the collection $B_1, B_2, \dots, B_m$ if $H$ contains at least one element from each $B_i$—that is, if $H \cap B_i$ is not empty for each $i$ (so $H$ "hits" all the sets $B_i$).

We now define the HITTING SET problem as follows. We are given a set $A = \{a_1, \dots, a_n\}$, a collection $B_1, B_2, \dots, B_m$ of subsets of $A$, and a number $k$. We are asked: Is there a hitting set $H \subseteq A$ for $B_1, B_2, \dots, B_m$ whose size is at most $k$?

Prove that HITTING SET is NP-complete.

**Question 3** (K&T 8.6)**.** Consider an instance of the SATISFIABILITY problem, specified by clauses $C_1, \dots, C_k$ over a set of Boolean variables $x_1, \dots, x_n$. We say that the instance is *monotone* if each term in each clause consists of a nonnegated variable; that is, each term is equal to $x_i$, for some $i$, rather than $\overline{x_i}$. Monotone instances of SATISFIABILITY are very easy to solve: they are always satisfiable, by setting each variable equal to 1.

For example, suppose we have the three clauses

$$(x_1 \vee x_2), (x_1 \vee x_3), (x_2 \vee x_3).$$

This is monotone, and indeed the assignment that sets all three variables to 1 satisfies all the clauses. But we can observe that this is not the only satisfying assignment: we could also have set $x_1$ and $x_2$ to 1, and $x_3$ to 0. Indeed, for any monotone instance, it is natural to ask how *few* variables we must set to 1 in order to satisfy it.

Given a monotone instance of SATISFIABILITY, together with a number $k$, the *Monotone Satisfiability with Few True Variables* problem asks: is there a satisfying assignment for the instance in which at most $k$ variables are set to 1? Prove that this problem is NP-complete.