Algorithms Activity 2: GCD analysis

Remember that these are not worksheets to be completed individually! You should work to answer the questions *as a team*.

Process Skills and Guided Inquiry

Although we will not use it every time, many of our class meetings will use an approach to learning called POGIL, which stands for Process-Oriented Guided Inquiry Learning. What does that mean? Let's start with "Guided Inquiry Learning".

Learning objective: Students will explain the benefits of a guided-inquiry approach to learning.

1 (Review) Brainstorm a list of as many details as you can remember from the class syllabus and academic integrity policy (don't peek!).

2 How much were you able to remember? What if the instructor had gone over the syllabus instead of having you answer questions about it as a team—do you think you would you have remembered more, less, or about the same?

3 This type of format, where you work together to answer questions that guide you through a learning process, is known as *guided inquiry*. What do you think are some of the benefits of guided inquiry (as compared to listening to a lecture)?

Don't just put what you think I want to hear—what do you honestly think are the benefits? If you don't think there are any benefits, go ahead and say that!

Learning objective: Students will identify process skills and explain how they are developed in a POGIL classroom.

4 (3 minutes) Imagine that your team is tasked with building a large, complex piece of software. There is no way you can do it by yourselves in any reasonable amount of time. Fortunately, you have a large budget, so you are going to hire people to help you. Brainstorm a list of qualities and skills you would look for in people that you hire.

5 (3 minutes) Now pick the three or four most important qualities/skills from your list and explain why they are important.

6 Most likely, some of the skills you identified were content-based and specific to computer science; whereas other skills are more general "soft" skills that have to do with the *process* of solving problems, being productive, and working well with other people. We call these *process skills*. In what ways, if any, do you think working on guided inquiry activities in a learning team with assigned roles will help you develop these skills? Which process skills will it help you develop?

Review questions

7 What is $27 \bmod 5$?

8 What is $2 \bmod 5$?

9 Which of the following statements is true, assuming that a and b are positive integers?

- $0 \le a \bmod b < b$
- $0 \le a \bmod b < a$

10 What is $5 \bmod 0$?

11 Is 0 divisible by 10?

Model 1: GCD

Definition 1. Recall that the *greatest common divisor*, or GCD, of two positive integers a and b is defined as the largest positive integer which evenly divides both a and b. The GCD of a and b is denoted $\gcd(a, b)$.

12 What is $\gcd(12, 30)$?

13 What are the prime factorizations of 12 and 30?

14 What do the prime factorizations of 12 and 30 have to do with $\gcd(12, 30)$?

15 What is $\gcd(144, 690)$?

16 What if we extend the definition of GCD to apply to all nonnegative integers? What should $\gcd(a, 0)$ be when $a > 0$?

Model 2: The Euclidean Algorithm

Consider the four algorithms specified below. They are all supposed to compute the GCD of nonnegative integers, but only two of them are correct.

GCDIa(m,n) =
  $a \leftarrow m$
  $b \leftarrow n$
  while $(a \neq 0)$
    if $a \leq b$
      then $b \leftarrow b \bmod a$
      else $a \leftarrow a \bmod b$
  if $a = 0$ **then return** b **else return** a

GCDIb(m,n) =
  $a \leftarrow m$
  $b \leftarrow n$
  while $(a \neq 0)$ **and** $(b \neq 0)$
    if $a \leq b$
      then $b \leftarrow b \bmod a$
      else $a \leftarrow a \bmod b$
  if $a = 0$ **then return** b **else return** a

GCDRa(a,b) =
  if $b = 0$
    then a
    else GCDRa(b, $a \bmod b$)

GCDRb(a,b) =
  if $b = 0$
    then a
    else GCDRb($a \bmod b$, b)

17 Trace the execution of each algorithm on the inputs $(144, 690)$.

18 In answering the previous question you should have found one of the incorrect algorithms. Figure out which of the other three is incorrect, and give example inputs for which it behaves incorrectly.

19 What do you think the I and R stand for in GCDI and GCDR?

20 For the correct algorithms, describe in a few sentences what happened to the values of a and b as the algorithm ran. Can you explain why the algorithms will always stop eventually?

21 Look at one of your execution traces from Question 17. Find the gcd of a and b after each iteration of the algorithm. What do you notice?