Algorithms Activity 6: Applications of BFS

BFS time

Recall from last class that we showed breadth-first search (BFS) can be implemented to run in $\Theta(|V| + |E|)$ time.

1 How small or big can $|E|$ be, relative to $|V|$?

(a) $|E|$ is $\Omega\Big(\qquad\qquad\Big)$ because ________________________________.

(b) $|E|$ is $O\Big(\qquad\qquad\Big)$ because ________________________________.

2 In terms of Θ, how fast does BFS run when G is a tree?

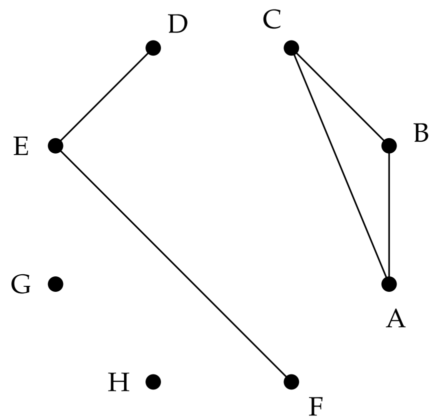
3 How fast does BFS run when G is very dense, *i.e.* it contains some constant fraction (say, half) of all possible edges?

A first application of BFS

4 Describe an algorithm to find the connected components of a graph G.

Input: a graph $G = (V, E)$
Output: a set of sets of vertices, `Set<Set<Vertex>>`, where each set contains the vertices in some (maximal) connected component. That is, all the vertices within each set should be connected; no vertex should be connected to vertices in any other sests; and every vertex in V should be contained in exactly one of the sets.

For example, given the graph below, the algorithm should return $\{\{D, E, F\}, \{C, B, A\}, \{G\}, \{H\}\}$.

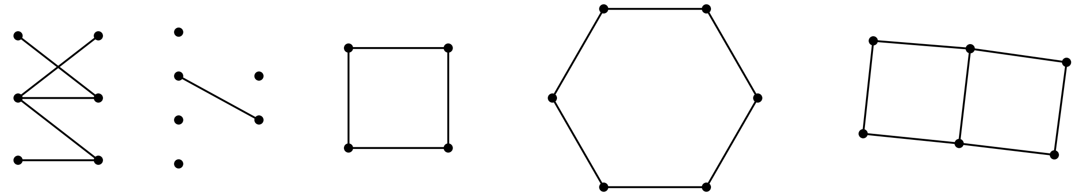


Describe your algorithm (using informal prose or pseudocode) and analyze its asymptotic running time.

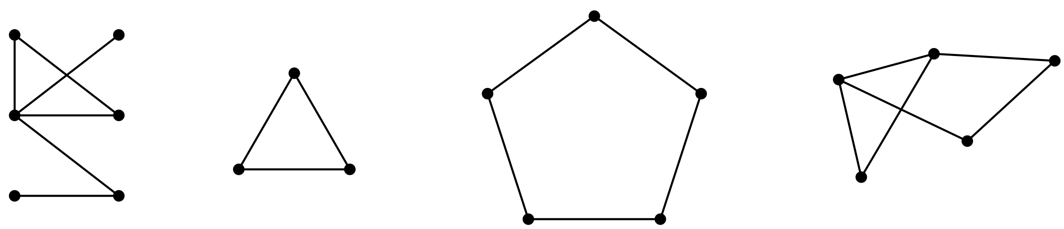
STOP

Model 1: Some graphs

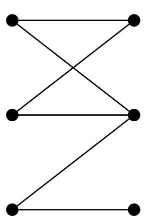
There is a mystery property X. Each graph either has property X or not.
These graphs have property X:

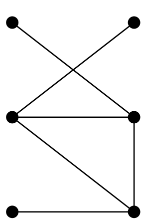


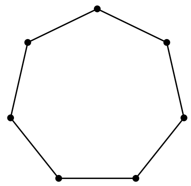
These graphs do not have property X:

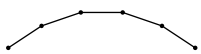


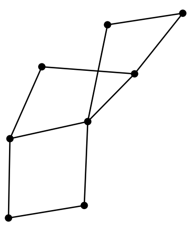
5 For each graph below, say whether you think it has property X.

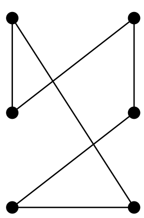
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

6 What do you think is the definition of property X?

7 Make a conjecture of the form: a graph G has property X if and only if G ____.