Algorithms Activity 7: Dijkstra's Algorithm

Model 1: Some graphs

Please copy the graphs from the board.

(Review) Begin by considering Graph 1.

1 What is the shortest path from *s* to *t*? How long is it?

2 What algorithm would you use to find it?

Now consider Graph 2. It looks just like Graph 1 except that each edge is labelled with a *weight*.

3 What do you think the "length" of a path means in this graph?

4 According to your definition of length, what is the shortest path from *s* to *t*? How long is it?

5 Why wouldn't your answer to Question 2 work here?

Now consider Graph 3. Imagine that each edge is a pipe that only allows water through in the direction the arrow is pointing. The number on the edge indicates how many seconds it takes for water to flow from one end of the pipe to the other. Now imagine that we hook up an (infinite) source of water to vertex *s* and watch the water start flowing through the network of pipes. Of course, water always flows in all possible directions. For example, as soon as we hook up the water source to vertex *s*, water immediately begins flowing along all three pipes leaving from *s*.

6 What is the first vertex (besides *s*) the water reaches? At what time does this happen (counting in seconds from the moment we hook up the water to vertex *s*)?

7 What is the second vertex the water reaches? At what time does it happen?

8 What is the third vertex the water reaches? At what time?

9 Suppose we changed the length of the pipe $s \to a$ from 15 to 20. How (if at all) would this change your answers to the previous questions?

10 Draw the situation after 32 seconds. (You may wish to draw directly on the graph in the model; or you can make a separate copy.) Which vertices has the water reached? Which pipes are full?

11 After 32 seconds, which vertex will the water reach next?

12 How does thinking about water flooding the graph help us solve the problem of finding shortest paths? Make a conjecture relating water in the graph to shortest paths between s and other vertices.