The first page of your homework submission must be a cover sheet answering the following questions. Do not leave it until the last minute; it's fine to fill out the cover sheet before you have completely finished the assignment. Assignments submitted without a cover sheet, or with a cover sheet obviously dashed off without much thought at the last minute, will not be graded.

- How many hours would you estimate that you spent on this assignment?

- Explain (in one or two sentences) one thing you learned through doing this assignment.

- What is one thing you think you need to review or study more? What do you plan to do about it?

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Question 1. Prove: for all $n \geq 1$, if G is a connected graph with n vertices and $n-1$ edges, then G has no cycles. (This is the third part of the proof from class, characterizing trees as having any two out of three properties.)

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Question 2. Let $G = (V, E)$ be an undirected graph with n vertices, with no self-loops (that is, no edges of the form (v, v) from a vertex to itself). Show that if every vertex has degree at least $n/2$, the graph is connected. If it makes your proof easier, you may assume that n is even.

Question 3. Consider the family of undirected graphs $\mathcal{H}_k$ defined as follows. $\mathcal{H}_k$ has 2^k vertices labelled with the integers 0 through $2^k - 1$. Vertices u and v are connected by an edge if and only if the binary representations of u and v differ in exactly one bit position. For example, in $\mathcal{H}_4$, the vertices 5 and 13 are connected by an edge since $5 = 0101_2$ and $13 = 1101_2$ differ in the first bit position, but the rest of the bits are the same.

Consider doing a BFS in $\mathcal{H}_{10}$ starting at node 0. How many vertices are in L_6, that is, the sixth layer generated by the BFS? Give your answer together with either a proof, or the program you used to calculate the answer. Either approach will receive full credit. (*Hint* if you choose to write a program: to flip the `j`th bit of an integer `n`, you can use `n ^ (1 << j)`, that is, the bitwise XOR of `n` with the result of shifting 1 left j times, that is, 2^j. These operators are valid syntax in many languages such as Java, Python, and C.)

Question 4. On the website you will find a file called `graph.txt` which describes a large undirected graph. The first line of the file contains a single integer which is the number of edges in the graph. Each subsequent line of the file describes one edge, and contains two space-separated strings which are the names of the two vertices at the endpoints of the edge.

Write a program (in a programming language of your choice) to find the shortest path from the vertex labelled with your first name[1] to the vertex labelled `END` (if there are multiple shortest paths you can find any one of them). You should submit a text file containing the list of vertices along this shortest path, starting with your name and ending with `END`. Each vertex should be on a separate line. For example, my solution looks like this:

```
Brent
ClTxyl
LPlAQf
fM4ZaY
hweBqP
qKykFp
VwZSvh
END
```

You should also turn in the code you used to find your path.

[1] Collisions are resolved by appending the first letter of your last name.

Note that you must submit your solution to this problem (including your code and the text file with your shortest path) **electronically**, even if you turn in the rest of the assignment on paper.