The first page of your homework submission must be a cover sheet answering the following questions. Do not leave it until the last minute; it's fine to fill out the cover sheet before you have completely finished the assignment. Assignments submitted without a cover sheet, or with a cover sheet obviously dashed off without much thought at the last minute, will not be graded.

- How many hours would you estimate that you spent on this assignment?

- Explain (in one or two sentences) one thing you learned through doing this assignment.

- What is one thing you think you need to review or study more? What do you plan to do about it?

Question 1. Explain how the Ford-Fulkerson algorithm can be implemented to run in $O(m \cdot \min(C_s, C_t))$ time (assuming that the edge capacities are all nonnegative integers), where m is the number of edges in the network, C_s is the total capacity of all edges leaving the source node s, and C_t is the total capacity of all edges entering the sink node t.

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Question 2. Let $G = (V, E)$ be a bipartite graph with $V = L \cup R$ (that is, every edge in G has one endpoint in L and one in R). A *matching* in G is a set of edges $M \subseteq E$ such that no two edges in M share an endpoint. A *maximum matching* is a matching with the greatest possible number of edges.

Design an algorithm to find a maximum matching in a given bipartite graph. Describe your algorithm, prove its correctness, and analyze its running time.

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Question 3 (K&T 7.6)**.** Consider a set of mobile computing clients in a certain town who each need to be connected to one of several possible *base stations*. We'll suppose there are n clients, with the position of each client specified by its (x, y) coordinates in the plane. There are also k base stations; the position of each of these is specified by (x, y) coordinates as well.

We wish to connect each client to exactly one of the base stations, but our choice of connections is constrained in the following ways. First, there is a *range parameter*, denoted by r: a client can only be connected to a base station that is within distance r. There is also a *load parameter* L: no more than L clients can be connected to any single base station.

Your goal is to design a polynomial-time algorithm for the following problem. Given the positions of a set of clients and a set of base stations, as well as the range and load parameters r and L, decide whether every client can be connected simultaneously to a base station, subject to the constraints r and L.

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